Cubic fourfolds, hyper-Kähler varieties and rationality problem

October 11-12, 2016, Lyon

Titles and Abstracts

Voisin: ***Decomposition of the diagonal and cubic hypersurfaces.***

In the first talk, I will describe various notions of decompositions of the diagonal, (and also the origin of the notion) with applications to algebraic cycles, coniveau, and stable rationality. The second talk will focus on the geometry of cubic hypersurfaces, where more can be said. In particular, there is only one notion of decomposition of the diagonal (cohomological and Chow-theoretic decompositions are equivalent). Furthermore, for cubic threefolds, the existence of the decomposition is equivalent to the algebraicity of the minimal class of the Jacobian, a very classical open problem. Throughout the lectures, I will put emphasis on open problems.

Colliot-Thélène: ***Non rationalité stable d'hypersurfaces cubiques sur des corps non algébriquement clos.***

On s'intéresse à la question de la rationalité stable, sur un corps de base F, des hypersurfaces cubiques lisses dans l'espace projectif de dimension n>3, lorsqu'elles possèdent un point F-rationnel. Pour F le corps des complexes, c'est une question ouverte. Pour F le corps des réels, on a des contre-exemples à la rationalité stable en toute dimension. Pour F un corps de fonctions de d variables sur le corps des complexes, on sait donner des contre-exemples à la rationalité stable lorsque n n'est pas trop grand par rapport à d. On décrira trois méthodes pour ce faire (l'une étant due à A. Chatzistamatiou et M. Levine).
Par ailleurs, sur le corps des complexes, en toute dimension, on montrera l'existence d'hypersurfaces cubiques lisse CH_0-universellement triviales.

Stellari: ***Bridgeland stability for semiorthogonal decompositions and cubic fourfolds.***

Various interesting results concerning the geometry of smooth cubic threefolds point to the relevance of constructing Bridgeland stability conditions on semiorthogonal decompositions. We illustrate some of them and a new method to induce stability conditions on semiorthogonal decompositions. We prove that it provides Bridgeland stability conditions on the Kuznetsov component of the derived category of cubic fourfolds. This is joint work in progress with Arend Bayer, Martí Lahoz and Emanuele Macrì.

Laza: ***Birational geometry of locally symmetric varieties of K3 type.***

The moduli spaces of polarized K3 surfaces (and other related objects, such as Hyperkähler manifolds, or cubic fourfolds) are quotients of Hermitian symmetric domains of Type IV by arithmetic groups. These local symmetric varieties are rich in special divisors (known as Heegner divisors or Noether-Lefschetz divisors) which essential control their birational geometry. In this talk, I will discuss the work of Looijenga, and the refinements of myself with O'Grady, on a directed variation of models for moduli spaces of K3 types. As application, I will discuss the case of moduli spaces of cubic fourfolds and degree 4 K3 surfaces.

Shen: ***Universal generation and rationality.***

In this talk, I will show that the Chow group of 1-cycles on a cubic hypersurface is universally generated by lines. In the case of cubic threefolds and cubic fourfolds, I will show how the existence of a decomposition of the diagonal can be translated into the truth of the integral Hodge conjecture for certain canonical classes.

Mboro: ***Decomposition of the diagonal for cubic threefolds over a field of positive characteristic***.

Adapting arguments of Voisin, we show that for a cubic threefold over an algebraically closed field of characteristic > 2, to admit a cohomological decomposition of the diagonal is equivalent to admit a Chow-theoretic decomposition of the diagonal. We deduce that such a cubic threefold admits a Chow-theoretic decomposition of the diagonal if and only if the minimal class of its intermediate Jacobian (i.e. the algebraic representative of the group of algebraically trivial 1-cycles of the cubic) is algebraic. Then, on the closure of a finite field of characteristic p > 2, the Tate conjecture for divisors on surfaces defined over finite fields of characteristic p predicts that any cubic threefold admits a Chow-theoretic decomposition of the diagonal.

Pacienza: ***0-cycles on some Calabi-Yau threefolds.***

I will report on a joint on-going project with Gilberto Bini and Robert Laterveer. In our work we provide a general criterion for Calabi-Yau 3-folds to verify a conjecture, due to Voisin and motivated by the Bloch-Beilinson conjecture, on 0-cycles on varietes with $p_g = 1$.Moreover we exhibit examples in which the criterion can effectively be applied.